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An Attractive Method for Displaying Material Damping Data

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This paper describes the development of a new reduced-temperature nomogram which greatly facilitates the display and correlation of complex modulus data for a linear thermorheologically simple viscoelastic damping material in such a way that the effects of frequency and temperature can be simultaneously taken into account. The method is based on the well-known temperature-frequency equivalence principle, which allows one to modify the frequency by a factor depending on temperature alone in such a way that complex modulus data points at a given frequency and temperature can be combined into a single set of curves, representing the loss factor and modulus as a function of a single variable, known as the reduced frequency. The superimposition of temperature isotherms completes the nomogram and thereby greatly expands the usefulness of the reduced-frequency graphs by allowing display on a single graph of complex modulus data at any frequency and temperature. This allows the possibility of generating and transmitting engineering data on viscoelastic material behavior to be used in many areas where such materials are being considered for vibration control. The nomogram is currently being used by industry and is being considered for national and international standards relating to display of engineering data for damping materials.

Nomenclature

a_{ij}	= parameters
$C_1, C_2...$	= parameters representing composition
E, E'	= real part of Young's modulus
E''	= imaginary part of Young's modulus
f	= frequency, Hz (also vector function)
i	= $\sqrt{-1}$ (also integer)
j	= integer
t	= time
T, T_i	= temperature (absolute)
T_0	= reference temperature (absolute)
α_T	= shift factor
ϵ_0	= maximum strain
ϵ	= strain
$\dot{\epsilon}$	= strain rate
ΔT	= temperature interval
η	= loss factor
ρ	= mass density
ρ_0	= mass density at reference temperature T_0
σ	= stress
σ_0	= maximum stress
Π_1, Π_2	= differential or integral operators
ω	= frequency (circular)

Introduction

THE problem of displaying, in a convenient and accurate manner, the linear complex modulus properties of polymeric or similar damping materials as a function of frequency and temperature is currently approaching critical proportions as industry begins to use such materials on a massive scale for noise and vibration control. Methods for displaying measured data, consisting usually of discrete sets of numerical data representing the modulus and loss factor values at a number of discrete frequencies and temperatures include plots of modulus and loss factor vs temperature at constant frequency, and vs frequency at constant temperature. Both display methods have limitations, especially if the data are not available at desired temperatures and frequencies. A more sophisticated technique is the reduced-frequency approach, due to Ferry and others,[1,2] which is essentially a plot of modulus and loss factor vs a parameter $f\alpha_T$, where f is the frequency and α_T is a nondimensional factor, called the shift factor, which depends only on the temperature for a given material composition. The method is not difficult to use, although somewhat tedious, both for generating the plots and in using them for design or other purposes. It is extremely useful, however, for collapsing the data onto two single curves of modulus and loss factor vs a single parameter, and seems well able to account for the idiosyncracies of many types of damping materials. The process cannot be simplified beyond a certain point, but is well adapted for computer operations. The greatest gain in convenience, once one has reached the point of having a reduced-frequency plot, comes from adding a set of additional lines that allow the user to directly read, at any given temperature and frequency, the desired modulus and loss factor values.[3] This is a great convenience for many purposes, as compared with the present situation in which one must read α_T from one graph or calculate it from a formula, then calculate $f\alpha_T$, and finally read the modulus and loss factor values from yet another graph. The net effect of this last improvement, which essentially creates a reduced-temperature nomogram, is that all available complex modulus data for each material are plotted on a single graph without any loss of data or any possibility of losing it. Furthermore, the nomogram does not in any way prevent one from taking advantage of yet another convenient design-oriented approach, which is to represent the data in terms of analytical functions of $f\alpha_T$ and of the temperature.[4,5]

The reduced-temperature nomogram, in this format, has been used extensively for 1) displaying the effect of composition changes on the damping behavior of polymers and vitreous enamels and 2) for displaying complex modulus data for many commercial polymeric damping materials for design applications. It is now under active consideration as American National and International Standards on the display of linear damping material behavior and has been computerized for ease of application. It will doubtless be well used in the future, and this paper will discuss the nomogram and these applications.

The Reduced-Frequency Concept

Physical Mechanisms of Damping

The phenomena which give rise to the energy dissipation in materials are very complex and depend on a large number of factors, in particular,

1) Internal factors: type of material, chemical composition, internal crystalline or noncrystalline structure, etc.

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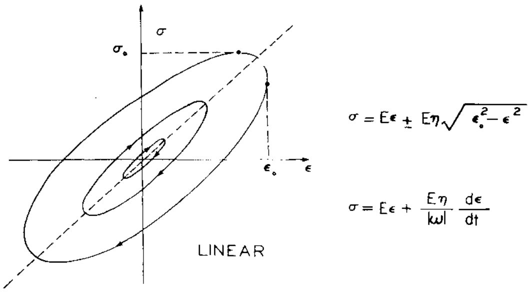

Fig. 1 Hysteresis loops.

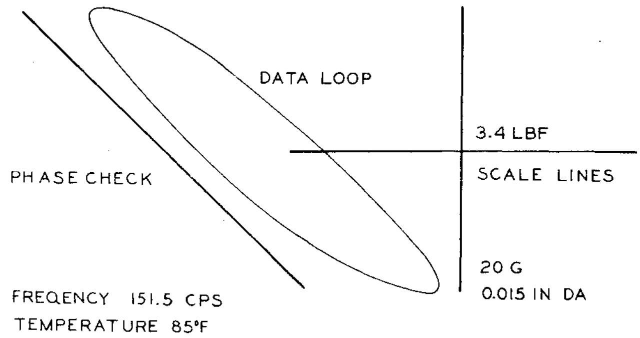

Fig. 2 Experimental hysteresis loop.

2) External factors: temperature, prestress, initial strain, etc.

3) Factors connected with motion: amplitude and frequency of deformation, state of stress, etc.

4) Specimen factors: geometry, scale, state of surfaces, bonding, etc.

One aim of an understanding of energy dissipation by damping is to predict the dynamic behavior of mechanical systems, by qualitatively and quantitatively describing the aforementioned factors in phenomenological terms, i.e., to introduce a reasonable mathematical model of the phenomena. Such a model should be, on the one hand, solvable (making use of computing facilities as necessary) and, on the other hand, the results of such a theoretical analysis should show sufficiently good agreement with experimental data to be useful.

Phenomenological Modeling of Cyclic Stress-Strain Behavior

The science of rheology gives the state equations based on thermodynamic laws of irreversible processes. The state equation for a material system can be written in the form

$$f[\Pi_1(\sigma),\Pi_2(\epsilon),t,T,C_1,C_2,\ldots]=0 \quad (1)$$

where f represents a vector function of variables, σ is the stress tensor, ϵ the strain tensor, t the time, T the temperature, and C_1, C_2... parameters depending on the physico-chemical properties of the medium and the external conditions. Π_1 and Π_2 represent differential or integral operators and are generally nonlinear. The state equations are essentially models of material behavior and, depending on the response to external excitation (external forces, variable temperatures, magnetic fields, chemical reactions, etc.), specify to some degree of approximation the materials. The essential elements of a model of a rheological medium are 1) elastic: work applied to changing volume or shape of a specimen is stored as potential energy, a reversible process; 2) viscous: resistance forces depending on velocity of deformation; and 3) plastic: dissipation of energy of the dry friction type, independent of velocity and time. The deformation processes in the viscous and plastic models are characterized by a particular relation between the work done and the deformation, and ways of realizing this deformation in time. The energy supplied is instantly and totally dissipated. A fourth medium should be mentioned, namely the rigid body. However, it is rarely applicable in the description of damping properties. The most general linear rheological model can be represented by the following relation:

$$\Pi_1(\sigma)=\Pi_2(\epsilon)$$

where Π_i are linear differential operators

$$\Pi_i=\sum_{j=0}^{n_i} a_{ij}\frac{\partial^j}{\partial t^j} \quad (2)$$

where $a_{ij}=$const, $i=1,2$. n_i can be a finite number (for lumped models of systems) or infinite (for continuous models). If the stresses and strains vary harmonically with time, $\sigma=\sigma_0 e^{i\omega t}$, $\epsilon=\epsilon_0 e^{i\omega t}$, this relationship between σ_0 and ϵ_0 takes the form:

$$\left(\sum_{j=0}^{n_1} a_{1j}(i\omega)^j\right)\sigma_0 e^{i\omega t}=\left(\sum_{j=0}^{n_2} a_{2j}(i\omega)^j\right)\epsilon_0 e^{i\omega t}$$

so that

$$\sigma_0=\epsilon_0\left(\sum_{j=0}^{n_2} a_{2j}(i\omega)^j \Big/ \sum_{j=0}^{n_1} a_{1j}(i\omega)^j\right) \quad (3)$$

The coefficients a_{ij} can be complex, so that, ultimately, one may write

$$\sigma_0=(E'+iE'')\epsilon_0 \quad (4)$$

where E' and E'' are real numbers which depend on frequency (and temperature). In terms of this model, one can represent the material behavior either through the many non-frequency-dependent coefficients a_{ij} or through the two frequency-dependent moduli E' and E''. It is often far more convenient, both for measurement and for analysis in the frequency domain, to work with the complex modulus $E^*=E'+iE''$ rather than with a large set of numbers a_{ij} which are difficult to measure on an individual basis. For analysis of linear viscoelastic material behavior in the frequency domain, therefore, the complex modulus representation is very popular and quite well accepted, despite its limitations for analysis of nonharmonic deformations. It is frequently more convenient to use the form

$$\sigma_0=E(1+i\eta)\epsilon_0 \quad (5)$$

instead of the form given in Eq. (4). Another method of representation, again for cyclic deformation, may be illustrated by rewriting Eq. (5) in a form appropriate for noncomplex notation. If we write $\epsilon=\epsilon_0\sin\omega t$, $\sigma=\sigma_0\sin(\omega t+\phi)$, Eq. (5) may be put in the form

$$\sigma(t)=E\epsilon(t)+(E\eta/|\omega|)\dot{\epsilon}(t) \quad (6)$$

The graph of σ vs ϵ is then a closed loop, called a hysteresis loop, as illustrated in Fig. 1. The loop is of elliptical shape for materials having properties describable by Eq. (6). The equation of this ellipse may be written

$$\sigma=E\epsilon\pm E\eta\epsilon_0\sqrt{1-\epsilon^2/\epsilon_0^2} \quad (7)$$

where σ_0 is the maximum stress and ϵ_0 the maximum strain. This apparently nonlinear hysteresis loop nevertheless

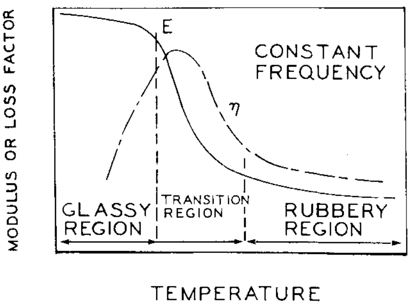

Fig. 3 Variation of complex modulus with temperature.

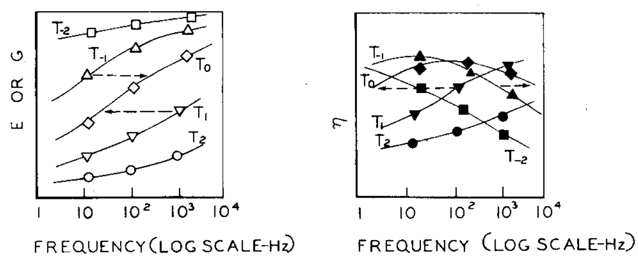

Fig. 4 Variation of E and η with frequency.

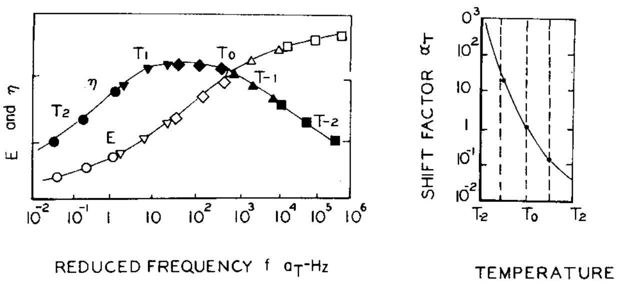

Fig. 5 Reduced-frequency plot.

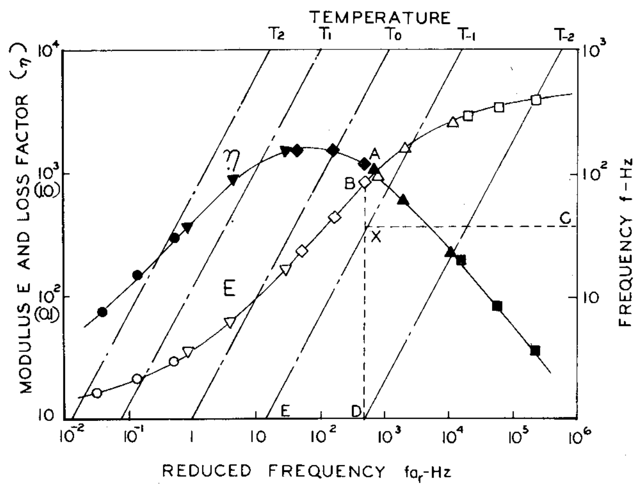

Fig. 6 Reduced-temperature nomogram.

corresponds to the linear relationship given by Eq. (6) and forms the basis for other methods of analysis more useful for truly nonlinear behavior.[6] Figure 2 shows a typical measured hysteresis loop for a linear damping material at a specific frequency and temperature.[7] It illustrates very well the viability of the assumption that the hysteresis loop is elliptical for linear viscoelastic materials.

Effects of Frequency and Temperature on Complex Modulus Properties of Linear Damping Materials

One of the ways in which damping materials differ from each other is in the variation of E and η with frequency, temperature, strain amplitude, and prestrain. Temperature is by far the most important factor, since E can in some cases vary by as many as five orders of magnitude over a narrow temperature range. The variation of Young's modulus E, or shear modulus G, and loss factor η with temperature at fixed frequency and at low cyclic strain amplitude are typically of the form shown in Fig. 3. Three distinct temperature regions are observed, namely the glassy, transition, and rubbery regions. In the glassy region E is high and η is low; in the transition region E varies rapidly with temperature and η is high; in the rubbery region E varies more slowly with temperature and η is lower than in the transition region, although not always as low as in the glassy region. At the very highest temperatures, irreversible thermal decomposition usually occurs.

Although the variation of complex moduli with frequency is less drastic than with temperature, it is important. For a typical material, graphs of E and η vs frequency at a number of temperatures might look like those in Fig. 4.

Temperature-Frequency Equivalence (Reduced Frequency)

If the effects of frequency and temperature on damping material behavior are to be taken into account simultaneously, one of the most useful techniques for presenting the experimental data is by means of the temperature-frequency equivalence principle (reduced frequency) for linear viscoelastic materials.[1,2] In this approach, E (or more exactly $T_0\rho_0 E/T\rho$) and η are plotted against a so-called reduced frequency $f\alpha_T$, where f is the actual frequency, α_T a function of absolute temperature T, and T_0 is a reference temperature, again on the absolute scale. Often, T_0/T and ρ_0/ρ, the density ratio, can be regarded as being approximately equal to unity over a wide temperature range and may therefore be ignored. The preparation of master curves of E and η vs $f\alpha_T$ is an extremely useful method of extrapolating and interpolating experimental data and for evaluating effects of composition changes on material behavior. It is also an extremely effective way of making maximum use of a limited set of data.

For example, in a test series, one may have data over a frequency range of 100-1000 Hz and a temperature range of 0-100°C and wish to estimate E and η at 50°C and 2 Hz. In order to create the master curve, one must seek a best fit of the data. This is most satisfactorily accomplished, at least initially, in an empirical manner by judging the factor α_T at each temperature on the basis of the shift needed to make the curve of $\log E$ vs log frequency at temperature T_i match that at temperature T_0, while at the same time matching the curves of $\log \eta$ vs log frequency at the same temperatures. T_0 can be initially selected arbitrarily. The outcome of this process is a set of curves such as those shown in Fig. 5.

Reduced-Temperature Nomogram

Basic Concept

The graph of $\log E$ and $\log \eta$ vs $f\alpha_T$ represents a fundamental relationship between the various parameters and variables. However, its use to read directly the numerical values of E and η for any given frequency and temperature is hampered by the minor but important inconvenience of having to read α_T off one graph, calculating $f\alpha_T$, and then reading E and η off another graph. Because of this, published data are often difficult to use because one of these graphs is omitted, usually that of α_T vs temperature.

If, however, we relabel the scales on this graph, we can readily create an extremely simple nomogram to do this tedious, if simple, calculation for us. We simply use the right-hand scale for frequency, as indicated in Fig. 6. Then for the horizontal line corresponding to $f = 1$ Hz, $f\alpha_T$ is equal to α_T, and points can be drawn corresponding to the values of α_T at selected temperatures. For example, the point E corresponds to the value of α_T at temperature T_{-1}. Similarly, for $f = 10$ Hz, $f = 10\alpha_T$ and so, again, points can be drawn in for each temperature. If this is done at many frequencies, the locus of points corresponding to each T_i is an oblique (sloping) line as shown in Fig. 6. To use the resulting nomogram, for each selected T and f, one simply moves up the oblique line corresponding to this temperature until it intersects the horizontal line corresponding to the selected frequency. The point of intersection X corresponds to the correct value of $f\alpha_T$, and one need only read vertically the values of E and η. Another possible use for such a nomogram is to reduce the data in the first instance.[3] For, if one selects the proper position for T_0 and the interval ΔT between T_0, T_1, T_2 ..., then the grid of lines so generated can be used to place the test data in position at the proper values of $f\alpha_T$. In effect one implies a certain variation of α_T with T by drawing such a nomogram, and only one combination of values of T_0 and ΔT will give an adequate reduction of the data.

Applications

This reduced-temperature nomogram concept has been used to reduce experimental data for many viscoelastic damping materials in the past several years.[8-10] A very interesting illustration of the method is given for

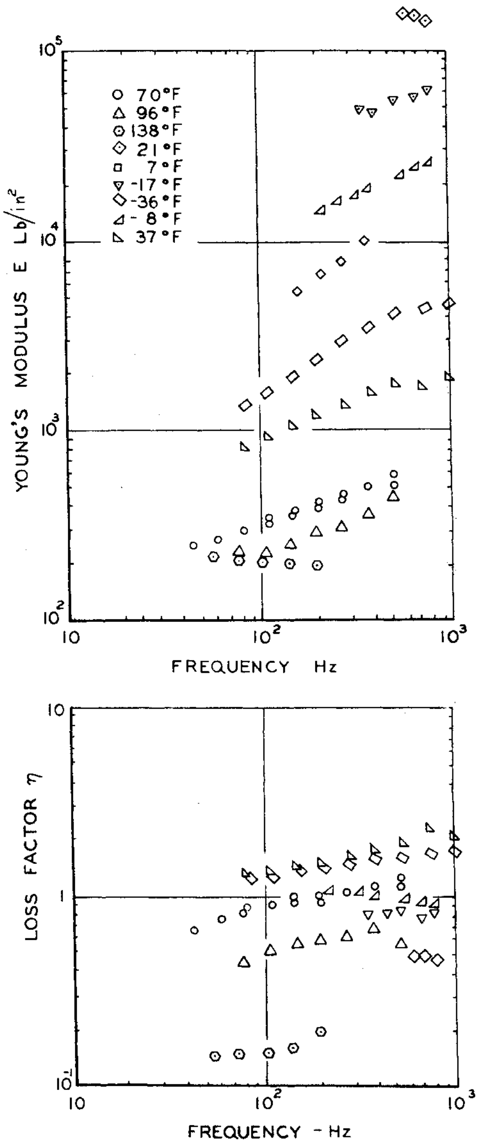

Fig. 7 *E* **and** η **vs frequency for polyisobutylene (PIB).**

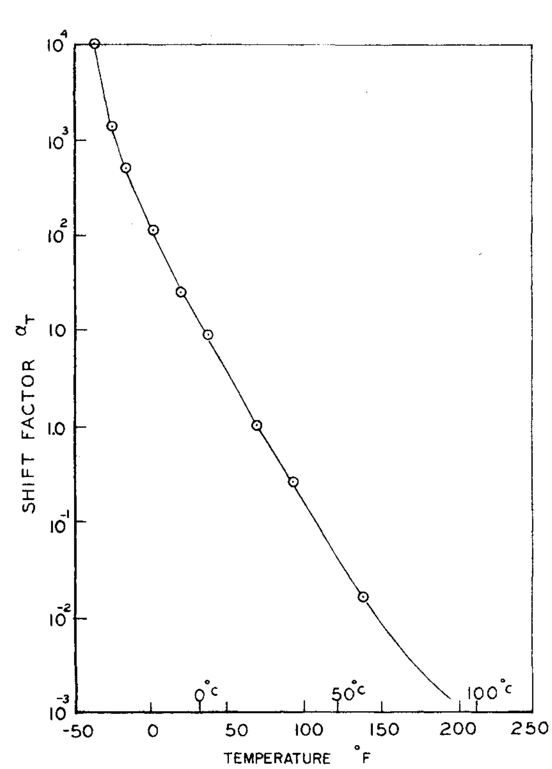

Fig. 8 α_T **vs temperature for PIB.**

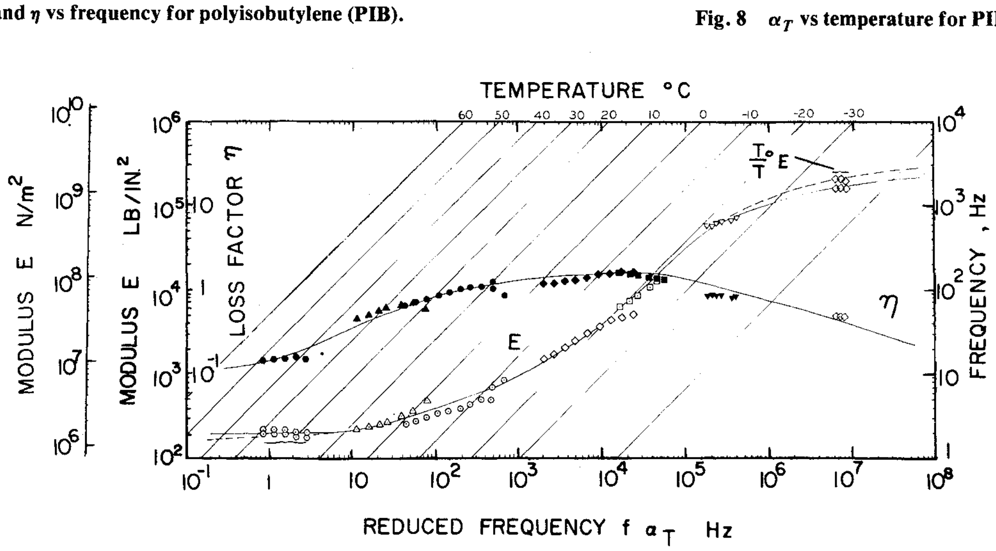

Fig. 9 Reduced-temperature nomogram for PIB.

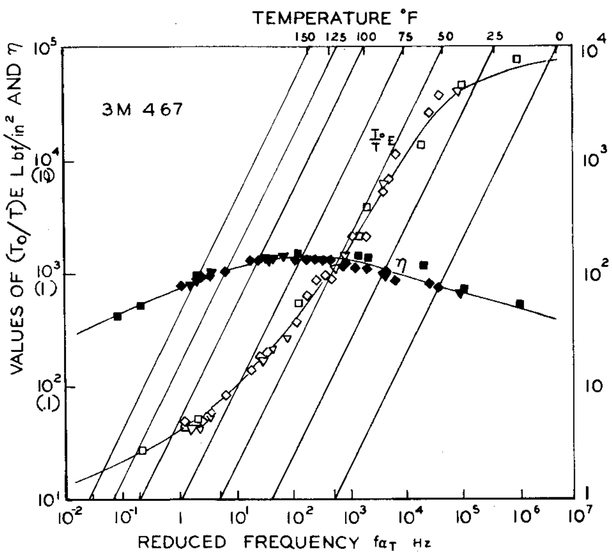

Fig. 10 Reduced-temperature nomogram for viscoelastic adhesive.

polyisobutylene (PIB), based on mechanical impedance measurements conducted some time ago.[11,12] The relevant test data are summarized in Fig. 7. Figure 7 shows the graphs of E and η plotted against frequency in the usual manner. From direct estimation of the horizontal shift needed to make the data at each temperature match, as closely as possible, that at an arbitrarily chosen reference temperature $T_0 = 70°F$ (21°C), one can construct a graph of α_T vs temperature as in Fig. 8. Figure 9 can then be drawn, showing E and η plotted against $f\alpha_T$. The graphs are quite smooth, as is usually the case when the initial data are accurate. The nomogram is now constructed by defining the frequency scale as 1-10^4 Hz on the right-hand side of Fig. 9, marking in the points corresponding to α_T at several temperatures along the horizontal line $f = 1$, and finally constructing the oblique temperature lines at unit slope from these points.

Figure 10 shows a nomogram for a typical viscoelastic adhesive.[13] The peak loss factor occurs in this instance around room temperature for the frequency range of common interest, 10^2-10^4 Hz. Figure 11 shows similar data for the same material, but plotted using a computer program for data reduction and plotting.[5,9] The process simply automates part

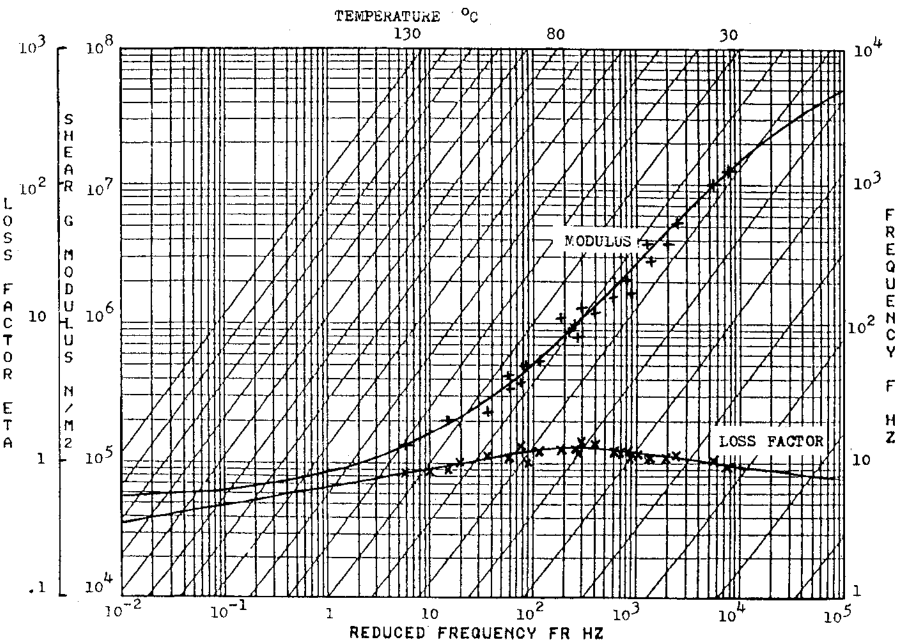

Fig. 11 Computer-generated nomogram (3M-467).

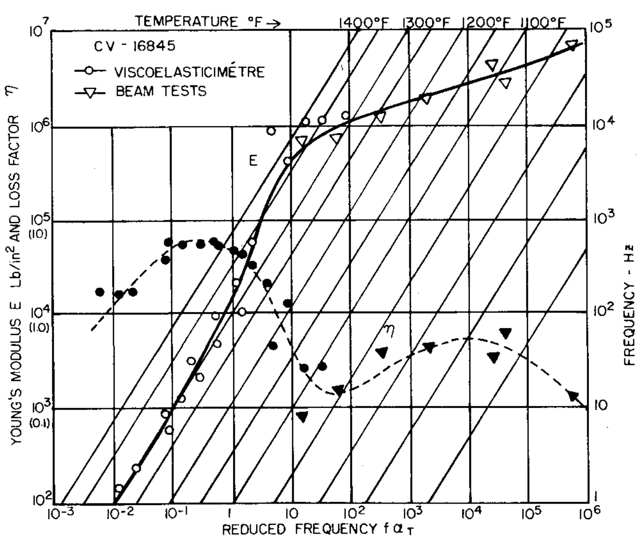

Fig. 12 Reduced-temperature nomogram for enamel (CV-16845).

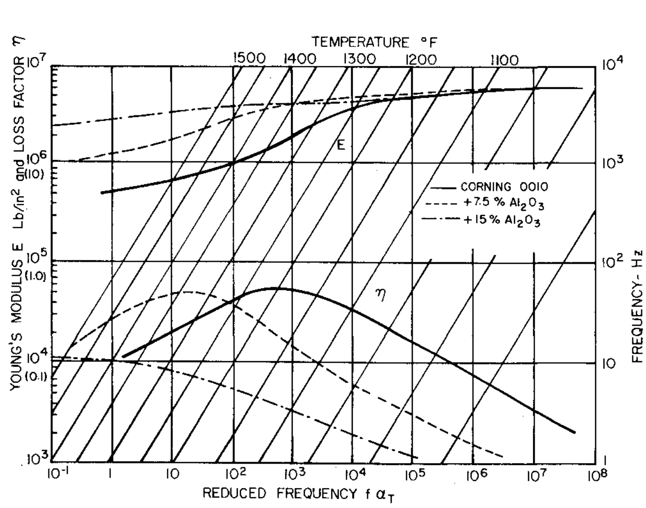

Fig. 13 Reduced-temperature nomogram showing effects of composition changes.

of the hand processing, but also uses a more formalized process and an algebraic equation to determine α_T as a function of temperature.

Figures 12 and 13 show reduced-temperature nomograms for some typical vitreous enamels used for high-temperature damping treatments.[14] In Fig. 12, the data from two different test methods (coated beam for high values of *E*, impedance method for low values of *E*) are compared for a commercial enamel. The two relaxation peaks are clearly seen. Figure 13 shows the effect of composition changes on the damping behavior of a particular glass in the vicinity of the lower temperature relaxation peak, as measured by coated beam tests. The effect of changing the amount of Al_2O_3 is seen to be to reduce the variation of *E* with temperature and to reduce the value of η. Many such tests have been conducted.[9,10]

The usefulness of the reduced-temperature nomogram is becoming very well accepted, and several current standards committees are striving to develop workable standards based on the concept. Specifically, Working Group S2-73 of the ANSI S2 Committee is working on the development of a standard entitled "Characterization and Graphical Representation of Linear Damping Material Properties,"[15] and Working Group 13 of ISO/TC-108 is developing an international standard on "Use of Materials for Damping of Vibrating Structures."[16] Similarly, ASTM Subcommittee E-33, Task Group E33.03-M is working on the development of a standard entitled "Standard for Measuring Damping Properties of Materials"[17] relating to measurement of viscoelastic damping material properties. The nomogram is to be used in the data reduction process for a round-robin series of tests to evaluate the standard.

Conclusions

The reduced-temperature nomogram has been described and shown to represent a simple and useful approach for describing linear complex modulus data as a function of frequency and temperature. Some current uses of the technique have been reviewed.

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[16] International Standards Organization Technical Committee TC-108, Working Group WG-13, Chairman, J. P. Henderson, Wright Aeronautical Laboratories, Wright-Patterson AFB, Ohio.

[17] American Society of Testing and Materials, Committee ASTM E-33, Task Group E-33.03-M, Secretary, W. J. Hanson, Liberty Mutual Insurance Co., Hopkinton, Mass.